

A_4 symmetry and lepton masses and mixing

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Abstract. Stimulated by Ma’s idea, which explains the tribimaximal neutrino mixing by assuming an A_4 flavor symmetry, a lepton mass matrix model is investigated. A Frogatt–Nielsen-type model is assumed, and the flavor structures of the masses and mixing are caused by the VEVs of $SU(2)_L$ singlet scalars ϕ_i^u and ϕ_i^d ($i = 1, 2, 3$), which are assigned to $\mathbf{3}$ and $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ of A_4 , respectively. Possible charged lepton and neutrino mass spectra and mixing are investigated.

1 Introduction

It is generally assumed that masses and mixings of the quarks and leptons will obey a simple law of nature. Then it is also likely that the masses and mixings of those fundamental particles will be governed by a symmetry. However, even if there is such a simple relation in the quark sector, it is hard to see such a relation in the quark sector, because the original symmetry will be spoiled by the gluon cloud. Therefore, in the present paper, we will confine ourselves to the investigation of the lepton masses and mixings.

It is well known that the observed neutrino mixing is nearly described by the so called tribimaximal mixing [1–10]:

$$U_{\text{TB}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

In order to understand the tribimaximal mixing, Ma [11] has recently proposed a neutrino mass matrix model based on a non-Abelian discrete symmetry A_4 . The symmetry A_4 seems to be very promising for a model of the leptons.

On the other hand, it is also well known that the observed charged lepton masses satisfy the relation [12–15]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \quad (2)$$

with remarkable precision. The mass formula (2) is invariant under any exchange $\sqrt{m_i} \leftrightarrow \sqrt{m_j}$ ($i, j = e, \mu, \tau$). This suggests that the lepton mass matrix model will be described by the S_3 permutation symmetry [16–19].

In order to understand (2), a seesaw-type mass matrix model [15, 20, 21] has been proposed:

$$M_e = m_L^e M_E^{-1} m_R^e. \quad (3)$$

Here, M_E is a mass matrix of hypothetical heavy leptons E_i ($i = 1, 2, 3$), and we have assumed $M_E \propto \mathbf{1} \equiv \text{diag}(1, 1, 1)$. The matrices m_L^e and m_R^e are mass matrices defined by $\bar{e}_L m_L^e E_R$ and $\bar{E}_L m_R^e e_R$, respectively, and we assume $m_L^e = m_R^e/k = y_e \text{diag}(v_1, v_2, v_3)$ (k is a constant), where v_i are vacuum expectation values (VEVs) of the three Higgs scalars $\phi_{Li} = (\phi_{Li}^\dagger, \phi_{Li}^0)$, and they satisfy the relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2. \quad (4)$$

The relation (4) can be derived from the following Higgs potential [22–24]:

$$V = \mu^2 (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3) + \frac{1}{2} \lambda_1 (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3)^2 + \lambda_2 (\phi_\sigma^\dagger \phi_\sigma) (\phi_\pi^\dagger \phi_\pi + \phi_\eta^\dagger \phi_\eta) + V_{\text{SB}}, \quad (5)$$

where ϕ_i ($i = 1, 2, 3$) are three objects of S_3 (fundamental basis), (ϕ_π, ϕ_η) and ϕ_σ are a doublet and singlet of S_3 , respectively, which are defined by

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \quad (6)$$

and V_{SB} is a soft symmetry breaking term [23, 24] that does not affect the derivation of the relation (4). The minimizing condition of the potential (5) leads to the VEV relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2. \quad (7)$$

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Equation (7) gives (4) because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2 \left(\frac{v_1 + v_2 + v_3}{\sqrt{3}} \right)^2. \quad (8)$$

(Note that although the Higgs potential (5) is invariant under the S_3 symmetry, it is not a general one of the S_3 -invariant form. As pointed out in [23, 24], a Higgs potential with the general form cannot lead to the relation (7). We need an additional requirement.) For a recent S_3 model of the lepton masses and mixings, see [25].

Considering the scenario for the charged lepton mass spectrum, the S_3 symmetry is also attractive, but, in the present paper, we will investigate an A_4 model by working with Ma's model [11] for the tribimaximal neutrino mixing. In the next section, we will show that the S_3 scenario for the charged lepton masses can be translated into the language of A_4 . In Sect. 3, we will give a Frogatt–Nielsen-type model [26] of the leptons based on an A_4 flavor symmetry. The mass matrix structures in the charged lepton and neutrino sectors are discussed in Sects. 4 and 5, respectively. In Sect. 6, a speculation about the neutrino masses will be given. In Sect. 7, a SUSY version of the Higgs potential (5) [see (24)] will be proposed. Finally, Sect. 8 is devoted to a summary. In order to obtain the tribimaximal mixing (1) and the charged lepton mass relation (2), we will need further phenomenological assumptions, (i) $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry and (ii) the universality of the coupling constants, in addition to the A_4 symmetry.

2 From S_3 into A_4

When we define $\bar{\psi} = (\bar{\psi}_1, \bar{\psi}_2, \bar{\psi}_3)$ and $\psi = (\psi_1, \psi_2, \psi_3)$ as $\mathbf{3}$ of A_4 , we can compose $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$ of A_4 as follows:

$$(\bar{\psi}\psi)_{\mathbf{1}} = \frac{1}{\sqrt{3}} (\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2 + \bar{\psi}_3\psi_3), \quad (9)$$

$$(\bar{\psi}\psi)_{\mathbf{1}'} = \frac{1}{\sqrt{3}} (\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2\omega + \bar{\psi}_3\psi_3\omega^2), \quad (10)$$

$$(\bar{\psi}\psi)_{\mathbf{1}''} = \frac{1}{\sqrt{3}} (\bar{\psi}_1\psi_1 + \bar{\psi}_2\psi_2\omega^2 + \bar{\psi}_3\psi_3\omega), \quad (11)$$

where

$$\omega = e^{i\frac{2}{3}\pi} = \frac{-1 + i\sqrt{3}}{2}. \quad (12)$$

The expressions (9)–(11) can be rewritten as

$$(\bar{\psi}\psi)_{\mathbf{1}} = (\bar{\psi}\psi)_\sigma, \quad (13)$$

$$(\bar{\psi}\psi)_{\mathbf{1}'} = \frac{1}{\sqrt{2}} [(\bar{\psi}\psi)_\eta - i(\bar{\psi}\psi)_\pi], \quad (14)$$

$$(\bar{\psi}\psi)_{\mathbf{1}''} = \frac{1}{\sqrt{2}} [(\bar{\psi}\psi)_\eta + i(\bar{\psi}\psi)_\pi], \quad (15)$$

where

$$\begin{pmatrix} (\bar{\psi}\psi)_\sigma \\ (\bar{\psi}\psi)_\eta \\ (\bar{\psi}\psi)_\pi \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \bar{\psi}_1\psi_1 \\ \bar{\psi}_2\psi_2 \\ \bar{\psi}_3\psi_3 \end{pmatrix}. \quad (16)$$

It is useful to define the following $(\phi_\sigma, \phi_\eta, \phi_\pi)$ basis correspondingly to (13)–(15):

$$\phi_{\mathbf{1}} = \phi_\sigma, \quad (17)$$

$$\phi_{\mathbf{1}'} = \frac{1}{\sqrt{2}}(\phi_\eta - i\phi_\pi), \quad (18)$$

$$\phi_{\mathbf{1}''} = \frac{1}{\sqrt{2}}(\phi_\eta + i\phi_\pi), \quad (19)$$

where the scalars $\phi_{\mathbf{1}}$, $\phi_{\mathbf{1}'}$ and $\phi_{\mathbf{1}''}$ are $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$ of A_4 . Then, A_4 -invariant Yukawa interactions that are composed of $\bar{\psi}$, ψ and ϕ are expressed as follows:

$$(\bar{\psi}\psi)_{\mathbf{1}}\phi_{\mathbf{1}} = (\bar{\psi}\psi)_\sigma\phi_\sigma, \quad (20)$$

$$\begin{aligned} (\bar{\psi}\psi)_{\mathbf{1}'}\phi_{\mathbf{1}''} &= \frac{1}{2} [(\bar{\psi}\psi)_\eta\phi_\eta + (\bar{\psi}\psi)_\pi\phi_\pi + i(\bar{\psi}\psi)_\eta\phi_\pi \\ &\quad - i(\bar{\psi}\psi)_\pi\phi_\eta], \end{aligned} \quad (21)$$

$$\begin{aligned} (\bar{\psi}\psi)_{\mathbf{1}''}\phi_{\mathbf{1}'} &= \frac{1}{2} [(\bar{\psi}\psi)_\eta\phi_\eta + (\bar{\psi}\psi)_\pi\phi_\pi - i(\bar{\psi}\psi)_\eta\phi_\pi \\ &\quad + i(\bar{\psi}\psi)_\pi\phi_\eta]. \end{aligned} \quad (22)$$

Hereafter, we will always assume a $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry, so that we obtain

$$(\bar{\psi}\psi)_{\mathbf{1}'}\phi_{\mathbf{1}''} + (\bar{\psi}\psi)_{\mathbf{1}''}\phi_{\mathbf{1}'} = (\bar{\psi}\psi)_\eta\phi_\eta + (\bar{\psi}\psi)_\pi\phi_\pi. \quad (23)$$

In the S_3 -invariant Higgs potential (5), the existence of the λ_2 term was essential for the derivation of the VEV relation (7). In the present A_4 model, if we adopt the basis $\phi = (\phi_\sigma, \phi_\eta, \phi_\pi)$, which is defined by (17)–(19), we can regard the Higgs potential (5) as an A_4 -invariant one:

$$\begin{aligned} V &= \mu^2 \left(\phi_{\mathbf{1}}^\dagger\phi_{\mathbf{1}} + \phi_{\mathbf{1}'}^\dagger\phi_{\mathbf{1}'} + \phi_{\mathbf{1}''}^\dagger\phi_{\mathbf{1}''} \right) \\ &\quad + \frac{1}{2}\lambda_1 \left(\phi_{\mathbf{1}}^\dagger\phi_{\mathbf{1}} + \phi_{\mathbf{1}'}^\dagger\phi_{\mathbf{1}'} + \phi_{\mathbf{1}''}^\dagger\phi_{\mathbf{1}''} \right)^2 \\ &\quad + \lambda_2 \left(\phi_{\mathbf{1}}^\dagger\phi_{\mathbf{1}} \right) \left(\phi_{\mathbf{1}'}^\dagger\phi_{\mathbf{1}'} + \phi_{\mathbf{1}''}^\dagger\phi_{\mathbf{1}''} \right) + V_{\text{SB}} \\ &= \mu^2 \left(\phi_\sigma^\dagger\phi_\sigma + \phi_\eta^\dagger\phi_\eta + \phi_\pi^\dagger\phi_\pi \right) \\ &\quad + \frac{1}{2}\lambda_1 \left(\phi_\sigma^\dagger\phi_\sigma + \phi_\eta^\dagger\phi_\eta + \phi_\pi^\dagger\phi_\pi \right)^2 \\ &\quad + \lambda_2 \left(\phi_\sigma^\dagger\phi_\sigma \right) \left(\phi_\eta^\dagger\phi_\eta + \phi_\pi^\dagger\phi_\pi \right) + V_{\text{SB}}. \end{aligned} \quad (24)$$

When we define (ϕ_1, ϕ_2, ϕ_3) by (6), we can obtain the VEV relation (4) for the VEVs $v_i = \langle \phi_i \rangle$, so that we obtain the charged lepton mass relation (2) from the A_4 -invariant Yukawa interaction:

$$\begin{aligned} (\bar{e}E)_{\mathbf{1}}\phi_{\mathbf{1}} + (\bar{e}E)_{\mathbf{1}'}\phi_{\mathbf{1}''} + (\bar{e}E)_{\mathbf{1}''}\phi_{\mathbf{1}'} \\ &= (\bar{e}E)_\sigma\phi_\sigma + (\bar{e}E)_\eta\phi_\eta + (\bar{e}E)_\pi\phi_\pi \\ &= \bar{e}_1 E_1 \phi_1 + \bar{e}_2 E_2 \phi_2 + \bar{e}_3 E_3 \phi_3, \end{aligned} \quad (25)$$

where e_L and E_R have been assigned to $\mathbf{3}$ of A_4 . (However, in the next section, we will not adopt the seesaw model (3), but we adopt a Frogatt–Nielsen-type model without the heavy leptons E_i .)

Note that, in the S_3 model, (ϕ_1, ϕ_2, ϕ_3) were three objects of S_3 and $(\phi_\sigma, \phi_\eta, \phi_\pi)$ were the singlet and doublet of S_3 , respectively, while, in the present A_4 model, $(\phi_\sigma, \phi_\eta, \phi_\pi)$ and (ϕ_1, ϕ_2, ϕ_3) are merely linear combinations of $(\phi_1, \phi_{1'}, \phi_{1''})$, and they are not irreducible representations of A_4 .

Thus, we have a possibility that we can build a model that leads not only to the tribimaximal mixing for the neutrinos, but also to the mass relation (2) for the charged leptons by developing Ma's idea.

3 Model

So far, we have considered three scalars to be ϕ_i $SU(2)$ doublets. However, such a model with multi-Higgs doublets causes a flavor changing neutral current (FCNC) problem. Therefore, in the present paper, we assume a Frogatt–Nielsen-type model [26]:

$$H_{\text{eff}} = y_e \bar{l}_L H_L^d \frac{\phi^d}{\Lambda_d} \frac{\phi^d}{\Lambda_d} e_R + y_\nu \bar{l}_L H_L^u \frac{\phi^u}{\Lambda_u} \nu_R + y_R \bar{\nu}_R \Phi \nu_R^*, \quad (26)$$

where the l_{iL} are $SU(2)_L$ doublet leptons $l_{iL} = (\nu_{iL}, e_{iL})$, H_L^d and H_L^u are conventional $SU(2)_L$ doublet Higgs scalars, ϕ^d and ϕ^u are $SU(2)_L$ singlet scalars, and Λ_d and Λ_u are scales of the effective theory. We consider $\langle \phi^f \rangle / \Lambda_f$ ($f = u, d$) to be of the order of 1. Here, we have not adopted the seesaw-type model (3) for the charged lepton sector, because the existence of $M_E \propto \mathbf{1}$ in (3) did not play any essential role in the flavor structure of the charged lepton mass matrix M_e . The scalar Φ has been introduced in order to generate the Majorana mass M_R of the right-handed neutrinos ν_R . The model is essentially unchanged compared with the seesaw model as far as the flavor structures are concerned. However, the scenario for the energy scale of the symmetry breaking is considerably changed, i.e. we assume that the VEVs of ϕ_i^f are of the order of the Planck mass scale although we have considered $\langle \phi_i \rangle \sim 10^2$ GeV in the seesaw model [27]. In other words, in the Frogatt–Nielsen-type model, the A_4 -broken structure of the effective Yukawa coupling constants is formed at the Planck mass scale. However, this is not a serious problem, because (1) is not so sensitive to the renormalization group equation (RGE) effects as far as the lepton sector is concerned [28, 29]. Although (2) is in remarkable agreement with the observed charged lepton masses (the pole masses), the standpoint in the present paper is that the remarkable coincidence is accidental and the relation (2) will be satisfied only approximately at a low energy scale.

Ma has assigned the scalars ϕ^d to $\mathbf{3}$ of A_4 in [11]. However, as we have shown in Sect. 2, since the scalars that can give the VEV relation (7) [or (4)] are not $\mathbf{3}$ of A_4 , but

Table 1. A_4 assignments of the fields

Fields	A_4	U(1)
l_L	$\mathbf{3}$	0
ν_R	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	0
e_R	$\mathbf{3}$	0
ϕ^u	$\mathbf{3}$	q_u
ϕ^d	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	q_d
Φ	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	0
H_L^u	$\mathbf{1}$	$-q_u$
H_L^d	$\mathbf{1}$	$-2q_d$

$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ of A_4 , we regard ϕ^d as $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ of A_4 in the present paper. Also, we will change the assignment of e_R and ν_R from those in the Ma model. Of course, the essential idea to obtain the tribimaximal mixing is indebted to the Ma model. The A_4 assignments in the present model are listed in Table 1. In order to forbid the unwelcome combinations $\bar{l}_L H_L^d (\phi^d)^n (\phi^u)^m e_R$ except for $(n, m) = (2, 0)$ and $\bar{l}_L H_L^u (\phi^d)^n (\phi^u)^m \nu_R$ except for $(n, m) = (0, 1)$, for example, we assume the following U(1) charge assignments: $Q(l_L) = Q(\nu_R) = Q(e_R) = Q(\Phi) = 0$, $Q(\phi^d) = -\frac{1}{2}Q(H_L^d) = q_d > 0$, and $Q(\phi^u) = -Q(H_L^u) = q_u > 0$, where $q_d/q_u \neq n/2$ and $q_u/q_d \neq n$ ($n = 0, 1, 2, 3, \dots$).

4 Charged lepton sector

In the charged lepton sector, the possible A_4 -invariant interactions $(\bar{e}_L e_R) \phi^d \phi^d$, i.e., $(\mathbf{3} \times \mathbf{3}) \times (\mathbf{1}, \mathbf{1}', \mathbf{1}'') \times (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, are given by

$$\begin{aligned} H_e &= (\bar{e}e)_1 [y_0 \phi_1 \phi_1 + y_1 (\phi_1' \phi_1'' + \phi_1'' \phi_1')] \\ &\quad + y_2 [(\bar{e}e)'_1 \phi_1' \phi_1' + (\bar{e}e)''_1 \phi_1'' \phi_1''] \\ &\quad + y_3 [(\bar{e}e)_{1'} (\phi_{1''} \phi_1 + \phi_1 \phi_{1''}) + (\bar{e}e)_{1''} (\phi_{1'} \phi_1 \\ &\quad + \phi_1 \phi_{1'})] \\ &= (\bar{e}e)_\sigma [y_0 \phi_\sigma^2 + y_1 (\phi_\pi^2 + \phi_\eta^2)] \\ &\quad + \frac{1}{\sqrt{2}} y_2 [(\bar{e}e)_\eta (\phi_\eta^2 - \phi_\pi^2) - 2(\bar{e}e)_\pi \phi_\eta \phi_\pi] \\ &\quad + 2y_3 [(\bar{e}e)_\eta \phi_\eta \phi_\sigma + (\bar{e}e)_\pi \phi_\pi \phi_\sigma], \end{aligned} \quad (27)$$

where, for convenience, we have dropped the index d , and we have assumed the $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry. Furthermore, if we assume universality of the coupling constants,

$$y_0 = y_1 = y_2 = y_3, \quad (28)$$

we obtain

$$\begin{aligned} H_e &= y_0 [(\bar{e}e)_\sigma (\phi_\sigma^2 + \phi_\pi^2 + \phi_\eta^2) \\ &\quad + \frac{1}{\sqrt{2}} (\bar{e}e)_\eta (\phi_\eta^2 - \phi_\pi^2 + 2\sqrt{2} \phi_\eta \phi_\sigma) + (\bar{e}e)_\pi \\ &\quad \times (-2\phi_\eta \phi_\pi + 2\sqrt{2} \phi_\pi \phi_\sigma)] \end{aligned}$$

$$\begin{aligned}
&= y_0 \left[(\bar{e}e)_\sigma (\phi_1^2 + \phi_2^2 + \phi_3^2) + \frac{1}{\sqrt{2}} (\bar{e}e)_\eta (2\phi_1^2 - \phi_2^2 - \phi_3^2) \right. \\
&\quad \left. + \sqrt{\frac{3}{2}} (\bar{e}e)_\pi (\phi_3^2 - \phi_2^2) \right] \\
&= \sqrt{3} y_0 (\bar{e}_1 e_1 \phi_1^2 + \bar{e}_2 e_2 \phi_2^2 + \bar{e}_3 e_3 \phi_3^2), \quad (29)
\end{aligned}$$

where ϕ_i ($i = 1, 2, 3$) are defined by (6). As we discussed in Sect. 2, since we can write the Higgs potential (5) for $\phi^d = (\phi_\pi^d, \phi_\eta^d, \phi_\sigma^d)$, we can obtain the VEV relation (7) [i.e. (4) for $v_i = \langle \phi_i^d \rangle$]. Therefore, from (29) and (4), we can obtain the charged lepton mass relation (2).

5 Neutrino sector

Since $\bar{\nu}_L \phi^u \nu_R \sim \mathbf{3} \times \mathbf{3} \times (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, the A_4 -invariant Yukawa interactions are as follows:

$$H_\nu = y_0' (\bar{\nu}_L \phi^u)_\sigma \nu_{R\sigma} + y_1' \left[(\bar{\nu}_L \phi^u)_\eta \nu_{R\eta} + (\bar{\nu}_L \phi^u)_\pi \nu_{R\pi} \right], \quad (30)$$

so that we obtain the mass matrix m_L^ν , which is defined by

$$(\bar{\nu}_1 \bar{\nu}_2 \bar{\nu}_3)_L m_L^\nu \begin{pmatrix} \nu_\sigma \\ \nu_\eta \\ \nu_\pi \end{pmatrix}_R, \quad (31)$$

as follows:

$$m_L^\nu = \begin{pmatrix} \frac{1}{\sqrt{3}} y_0' v_1^u & y_1' \frac{2}{\sqrt{6}} v_1^u & 0 \\ \frac{1}{\sqrt{3}} y_0' v_2^u & -y_1' \frac{1}{\sqrt{6}} v_2^u & -y_1' \frac{1}{\sqrt{2}} v_2^u \\ \frac{1}{\sqrt{3}} y_0' v_3^u & -y_1' \frac{1}{\sqrt{6}} v_3^u & y_1' \frac{1}{\sqrt{2}} v_3^u \end{pmatrix}. \quad (32)$$

When we again assume universality of the coupling constants, $y_0' = y_1'$, we obtain

$$m_L^\nu = D U_{\text{TB}}, \quad (33)$$

$$D = y_0' \text{diag}(v_1^u, v_2^u, v_3^u), \quad (34)$$

and U_{TB} is the tribimaximal mixing matrix (1), where we have changed the basis of ν_R from $(\nu_\sigma, \nu_\eta, \nu_\pi)_R$ to $(\nu_\eta, \nu_\sigma, \nu_\pi)_R$.

Although the VEVs of the scalars $\phi^d = (\phi_\pi^d, \phi_\eta^d, \phi_\sigma^d)$ satisfy the VEV relation (7), the VEVs $v_i^u = \langle \phi_i^u \rangle$ do not have such a relation, because we cannot write an A_4 -invariant term which corresponds to the λ_2 term, i.e. $\phi_\sigma^2 (\phi_\pi^2 + \phi_\eta^2)$. The potential for the scalars $\phi^u = (\phi_1^u, \phi_2^u, \phi_3^u)$ is symmetric for any exchange $\phi_i^u \leftrightarrow \phi_j^u$. Therefore, we consider

$$\langle \phi_1^u \rangle = \langle \phi_2^u \rangle = \langle \phi_3^u \rangle \equiv v_u. \quad (35)$$

Then the mass matrix m_L^ν is diagonalized as

$$U_{\text{TB}}^T m_L^\nu = y_0' v_u \mathbf{1}. \quad (36)$$

If the Majorana mass matrix M_R is diagonal in the basis $(\nu_\eta, \nu_\sigma, \nu_\pi)_R$, i.e.

$$M_R = \text{diag}(M_\eta, M_\sigma, M_\pi), \quad (37)$$

we obtain the mixing matrix U_{MNS} and the eigenvalues $m_{\nu i}$ of the neutrino mass matrix $M_\nu = m_L^\nu M_R^{-1} (m_L^\nu)^T$,

$$U_{\text{MNS}} = U_{\text{TB}}, \quad (38)$$

$$m_{\nu 1} = (y_0' v_u)^2 \frac{1}{M_\eta}, \quad m_{\nu 2} = (y_0' v_u)^2 \frac{1}{M_\sigma},$$

$$m_{\nu 3} = (y_0' v_u)^2 \frac{1}{M_\pi}. \quad (39)$$

The explicit structure of $M_R = \text{diag}(M_\eta, M_\sigma, M_\pi)$ will be discussed in the next section.

6 Speculation on the neutrino mass spectrum

In order to speculate on the neutrino mass spectrum, let us assume that the Majorana masses are generated by the following interaction with the scalars $\Phi = (\Phi_1, \Phi_{1'}, \Phi_{1''})$:

$$\begin{aligned}
H_R &= [y_0^R \bar{\nu}_1 \nu_1^* + y_1^R (\bar{\nu}_{1'} \nu_{1''}^* + \bar{\nu}_{1''} \nu_{1'}^*)] \Phi_1 \\
&\quad + y_2^R (\bar{\nu}_{1'} \nu_{1'}^* \Phi_{1'} + \bar{\nu}_{1''} \nu_{1''}^* \Phi_{1''}) \\
&= [y_0^R \bar{\nu}_\sigma \nu_\sigma^* + y_1^R (\bar{\nu}_\pi \nu_\pi^* + \bar{\nu}_\eta \nu_\eta^*)] \Phi_\sigma \\
&\quad + \frac{1}{\sqrt{2}} y_2^R [(\bar{\nu}_\eta \nu_\eta^* - \bar{\nu}_\pi \nu_\pi^*) \Phi_\eta - (\bar{\nu}_\pi \nu_\eta^* + \bar{\nu}_\eta \nu_\pi^*) \Phi_\pi], \quad (40)
\end{aligned}$$

where, for convenience, the notation $\nu_{1'}^*$ denotes $\nu_{1'}^* = (\nu^*)_{1'}$ [not $\nu_{1'}^* = (\nu_{1'})^*$, so that $(\nu_{1'}^*, \nu_{1''}^*, \nu_{1''}^*)$ are transformed as $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ [not $(\mathbf{1}, \mathbf{1}'', \mathbf{1}')$] under the A_4 symmetry. We assume that the VEVs of $\Phi \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ satisfy the same relation as (7) for ϕ^d

$$\langle \Phi_\pi \rangle^2 + \langle \Phi_\eta \rangle^2 = \langle \Phi_\sigma \rangle^2, \quad (41)$$

because ϕ^d and Φ are assigned to the same multiplets $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. However, if we consider $\langle \Phi_\pi \rangle \neq 0$, the matrix M_R cannot become diagonal. Therefore, we assume $\langle \Phi_\pi \rangle = 0$, so that we will take

$$\langle \Phi_\pi \rangle = 0, \quad \langle \Phi_\eta \rangle = \langle \Phi_\sigma \rangle. \quad (42)$$

This assumption corresponds to the assumption, although we have already assumed the $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry, that this symmetry holds for the VEV values of Φ , not for the fields, i.e. $\langle \Phi_{1'} \rangle = \langle \Phi_{1''} \rangle = \langle \Phi_\eta \rangle / \sqrt{2}$. Then we obtain the eigenvalues of M_R as follows:

$$\begin{aligned}
M_\eta &= \left(y_1^R + \frac{1}{\sqrt{2}} y_2^R \right) \langle \Phi_\sigma \rangle, \quad M_\sigma = y_0^R \langle \Phi_\sigma \rangle, \\
M_\pi &= \left(y_1^R - \frac{1}{\sqrt{2}} y_2^R \right) \langle \Phi_\sigma \rangle. \quad (43)
\end{aligned}$$

In order to speculate on the neutrino masses $m_{\nu i}$, we must reduce the number of parameters more. Therefore, let us assume that the fermion terms that couple to the scalars Φ_σ , Φ_η and Φ_π are normalized as

$$\begin{aligned}
H_R &= y_R \left[\left(\sin \alpha \bar{\nu}_\sigma \nu_\sigma^* + \cos \alpha \frac{\bar{\nu}_\pi \nu_\pi^* + \bar{\nu}_\eta \nu_\eta^*}{\sqrt{2}} \right) \Phi_\sigma \right. \\
&\quad \left. + \frac{\bar{\nu}_\eta \nu_\eta^* - \bar{\nu}_\pi \nu_\pi^*}{\sqrt{2}} \Phi_\eta - \frac{\bar{\nu}_\pi \nu_\eta^* + \bar{\nu}_\eta \nu_\pi^*}{\sqrt{2}} \Phi_\pi \right]. \quad (44)
\end{aligned}$$

Then we can write the eigenvalues (43) as follows:

$$\begin{aligned} M_\eta &= \frac{y_R}{\sqrt{2}} \langle \Phi_\sigma \rangle (1 + \cos \alpha), & M_\sigma &= y_R \langle \Phi_\sigma \rangle, \\ |M_\pi| &= \frac{y_R}{\sqrt{2}} \langle \Phi_\sigma \rangle (1 - \cos \alpha), \end{aligned} \quad (45)$$

so that we obtain the neutrino mass spectrum

$$\begin{aligned} m_{\nu 1} &= \frac{1}{1 + \cos \alpha} m_{\nu 0}, & m_{\nu 2} &= \frac{1}{\sqrt{2} \sin \alpha} m_{\nu 0}, \\ m_{\nu 3} &= \frac{1}{1 - \cos \alpha} m_{\nu 0}, \end{aligned} \quad (46)$$

where $m_{\nu 0} = (y_0' v_u)^2 / (y_R \langle \Phi_\sigma \rangle / \sqrt{2})^2$. For the observed ratio [30–33]

$$\begin{aligned} R_{\text{obs}} &\equiv \frac{\Delta m_{\text{solar}}^2}{\Delta m_{\text{atm}}^2} = \frac{(7.9_{-0.5}^{+0.6}) \times 10^{-5} \text{ eV}^2}{(2.74_{-0.26}^{+0.44}) \times 10^{-3} \text{ eV}^2} \\ &= (2.9 \pm 0.5) \times 10^{-2}, \end{aligned} \quad (47)$$

we obtain the predicted ratio

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{m_{\nu 2}^2 - m_{\nu 1}^2}{m_{\nu 3}^2 - m_{\nu 2}^2} = \frac{(3 \cos \alpha - 1)(1 - \cos \alpha)}{(3 \cos \alpha + 1)(1 + \cos \alpha)}. \quad (48)$$

For example, for $\alpha = \pi/6$, we obtain

$$R = \frac{(3\sqrt{2} - 2)(2 - \sqrt{3})}{(3\sqrt{2} + 2)(2 + \sqrt{3})} = 0.0319. \quad (49)$$

The value is in good agreement with the observed value (47). By putting $m_{\nu 3} = \sqrt{\Delta m_{\text{atm}}^2}$, we obtain $m_{\nu 1} = (0.38 \pm 0.02) \times 10^{-2} \text{ eV}$, $m_{\nu 2} = (0.99_{-0.05}^{+0.08}) \times 10^{-2} \text{ eV}$, and $m_{\nu 3} = (5.23_{-0.25}^{+0.40}) \times 10^{-2} \text{ eV}$.

However, the theoretical reason for $\alpha = \pi/6$ is unclear. Since we have assumed universality of the coupling constants in the interactions (27) and (31), rather, the case with the universal coupling $y_0^R = y_1^R$,

$$\begin{aligned} H_R &= y_R \left(\frac{\bar{\nu}_\sigma \nu_\sigma^* + \bar{\nu}_\pi \nu_\pi^* + \bar{\nu}_\eta \nu_\eta^*}{\sqrt{3}} \Phi_\sigma + \frac{\bar{\nu}_\eta \nu_\eta^* - \bar{\nu}_\pi \nu_\pi^*}{\sqrt{2}} \Phi_\eta \right. \\ &\quad \left. - \frac{\bar{\nu}_\pi \nu_\pi^* + \bar{\nu}_\eta \nu_\eta^*}{\sqrt{2}} \Phi_\pi \right), \end{aligned} \quad (50)$$

is likely. This case corresponds to $\cos \alpha = \sqrt{2/3}$, and it predicts

$$R = \frac{4\sqrt{6} - 9}{4\sqrt{6} + 9} = 0.0424. \quad (51)$$

The value (51) is somewhat large compared with the observed value (47), but, at present, this case cannot be ruled out within three sigma. Again, by putting $m_{\nu 3} = \sqrt{\Delta m_{\text{atm}}^2}$, we obtain

$$\begin{aligned} m_{\nu 1} &= (0.53_{-0.03}^{+0.04}) \times 10^{-2} \text{ eV}, \\ m_{\nu 2} &= (1.17_{-0.05}^{+0.08}) \times 10^{-2} \text{ eV}, \\ m_{\nu 3} &= (5.23_{-0.25}^{+0.40}) \times 10^{-2} \text{ eV}. \end{aligned} \quad (52)$$

Anyhow, the predicted value of $m_{\nu 1}$ in the present A₄ model is relatively large compared with that in the S₃ model [25]. We eagerly expect the detection from future double beta experiments.

7 Superpotential for three flavor scalars

So far, we have not considered the supersymmetric version of the present model. Recently, Ma has proposed a SUSY version [34] of the Higgs potential (5) that can lead to the VEV relation $v_\pi^2 + v_\eta^2 = v_\sigma^2$, (7). In a similar way, we can write the superpotential W for the superfields $\phi^d = (\phi_{\mathbf{1}}^d, \phi_{\mathbf{1}'}^d, \phi_{\mathbf{1}''}^d)$ (hereafter, for convenience, we will drop the index d) by assuming the following.

1. The field ϕ_a ($a = \mathbf{1}, \mathbf{1}', \mathbf{1}''$) to the n th power, $(\phi_a)^n$ ($n = 1, 2, 3$), appears always accompanied with the factor $1/n!$ in the superpotential W .
2. In order to forbid unwelcome A₄-invariant terms, we require that W is invariant under the transformation

$$\phi_{\mathbf{1}'} \rightarrow -\phi_{\mathbf{1}'}, \quad \phi_{\mathbf{1}''} \rightarrow -\phi_{\mathbf{1}''}. \quad (53)$$

Under this requirement, the terms $(\phi_{\mathbf{1}'})^3$ and $(\phi_{\mathbf{1}''})^3$ are forbidden, but $\phi_{\mathbf{1}'} \phi_{\mathbf{1}''}$, $(\phi_{\mathbf{1}'})^2$ and $(\phi_{\mathbf{1}''})^2$ are not forbidden.

3. The A₄ symmetry is softly broken by a term W_{SB} .

As a result, we obtain the superpotential

$$\begin{aligned} W &= m \left(\phi_{\mathbf{1}'} \phi_{\mathbf{1}''} + \frac{1}{2!} \phi_{\mathbf{1}}^2 \right) + \lambda \left(\phi_{\mathbf{1}} \phi_{\mathbf{1}'} \phi_{\mathbf{1}''} + \frac{1}{3!} \phi_{\mathbf{1}}^3 \right) \\ &\quad + W_{\text{SB}}, \end{aligned} \quad (54)$$

$$W_{\text{SB}} = \varepsilon m \left(-\phi_{\mathbf{1}'} \phi_{\mathbf{1}''} + \frac{1}{2!} e^{i\theta} \phi_{\mathbf{1}'}^2 + \frac{1}{2!} e^{-i\theta} \phi_{\mathbf{1}''}^2 \right). \quad (55)$$

Here, although the first and second terms in W_{SB} do not break the A₄ symmetry, we have added those to W_{SB} in order to keep the result (7) independent of W_{SB} . We will show below that the superpotential (54) can lead to the VEV relation (7) independently of W_{SB} and the parameter θ in W_{SB} determines the ratio v_π/v_η , so that the charged lepton mass spectrum is completely determined only by the parameter θ .

The superpotential (54) can be rewritten in terms of the superfields $(\phi_\pi, \phi_\eta, \phi_\sigma)$ defined by (17)–(19) as follows:

$$\begin{aligned} W &= \frac{1}{2} m (1 - \varepsilon) (\phi_\eta^2 + \phi_\pi^2) + \frac{1}{2} m \phi_\sigma^2 \\ &\quad + \frac{1}{4} \varepsilon m e^{i\theta} (\phi_\eta^2 - \phi_\pi^2 - 2i \phi_\eta \phi_\pi) \\ &\quad + \frac{1}{4} \varepsilon m e^{-i\theta} (\phi_\eta^2 - \phi_\pi^2 + 2i \phi_\eta \phi_\pi) \\ &\quad + \frac{1}{2} \lambda \phi_\sigma \left(\phi_\eta^2 + \phi_\pi^2 + \frac{1}{3} \phi_\sigma^2 \right). \end{aligned} \quad (56)$$

Since

$$\frac{\partial W}{\partial \phi_\pi} = [m(1 - \varepsilon) + \lambda \phi_\sigma - 2\varepsilon m \cos \theta] \phi_\pi + 2\varepsilon m \sin \theta \phi_\eta, \quad (57)$$

$$\frac{\partial W}{\partial \phi_\eta} = [m(1 - \varepsilon) + \lambda \phi_\sigma + 2\varepsilon m \cos \theta] \phi_\eta + 2\varepsilon m \sin \theta \phi_\pi, \quad (58)$$

$$\frac{\partial W}{\partial \phi_\sigma} = m \phi_\sigma + \frac{1}{2} \lambda (\phi_\eta^2 + \phi_\pi^2 + \phi_\sigma^2), \quad (59)$$

the minimization conditions of the potential lead to the relations

$$\tan \theta = \frac{2v_\eta v_\pi}{v_\eta^2 - v_\pi^2}, \quad (60)$$

$$v_\pi^2 + v_\eta^2 = v_\sigma^2, \quad (61)$$

$$m + \lambda v_\sigma = 0. \quad (62)$$

Note that the derivation of the relation (60) is independent of the explicit values of m , λ and ε , and the derivation of the relation (61) is independent of the explicit values of m , λ , ε and θ . [Also note that the conditions (57)–(59) can lead to an alternative solution with $(1 - 2\varepsilon)(v_\pi^2 + v_\eta^2) = (1 + 2\varepsilon)v_\sigma^2$ and $(1 - 2\varepsilon)m + \lambda \phi_\sigma = 0$ instead of (61) and (62), respectively. However, we have taken only the solution that is independent of the parameter ε .] Thus, without spoiling (7) [see (61)], we have been able to choose the symmetry breaking term W_{SB} that fixes the VEV ratio v_π/v_η .

From the observed values [35] of the charged lepton masses, we obtain the numerical values $z_1 = 0.016473$, $z_2 = 0.236869$ and $z_3 = 0.971402$, where the parameters z_i are defined by $\sqrt{m_{ei}} = z_i v_d$ with $z_1^2 + z_2^2 + z_3^2 = 1$, so that, for the VEVs of ϕ_a defined by (6), we obtain $z_\pi = 0.519393$, $z_\eta = -0.479824$ and $z_\sigma = 0.707106$. Therefore, we can obtain the value of θ as follows:

$$\tan \frac{\theta}{2} = \frac{v_\pi}{v_\eta} = \sqrt{3} \frac{z_3 - z_2}{2z_1 - z_2 - z_3} = -1.082466, \quad (63)$$

i.e. $\theta = -94.5354^\circ$. When we express the parameter z_i as

$$\begin{aligned} z_1 &= \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin \xi_e, \\ z_2 &= \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin \left(\xi_e + \frac{2}{3} \pi \right), \\ z_3 &= \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} \sin \left(\xi_e + \frac{4}{3} \pi \right), \end{aligned} \quad (64)$$

the angle θ is related to the parameter ξ_e by

$$\frac{\theta}{2} = \xi_e - \frac{\pi}{2}. \quad (65)$$

Note that the model gives $m_e \rightarrow 0$ in the limit of $\theta \rightarrow -\pi/2$.

8 Summary

In conclusion, on the basis of the A_4 symmetry, we have investigated a Frogatt–Nielsen-type model (26). The Higgs potential (24) for the scalars ϕ_i^d , which are assigned to $(\phi_1, \phi_{1'}, \phi_{1''})$ of A_4 , can lead to the VEV relation (7), $v_\pi^2 + v_\eta^2 = v_\sigma^2$, i.e. to the relation (4) for the VEVs $\langle \phi_i^d \rangle$ defined

in (6). Since the charged lepton interactions $\bar{\ell}_L H_L^d \phi^d e_R$ give $m_{ei} \propto \langle \phi_i^d \rangle^2$, we have obtained the charged lepton mass relation (2). For the neutrino sector, we have obtained the tribimaximal mixing (1) by assuming $\mathbf{3}$ of A_4 for the scalars ϕ^u .

However, it should be noted that, in order to obtain the above results, we have needed to assume the following requirements in addition to the A_4 symmetry: (i) the $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry and (ii) the universality of the coupling constants. Especially, in order to obtain the VEV relation (7) from the A_4 -invariant superpotential, the assumptions (i) and (ii) required in the previous section are required in addition to the $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry. Those assumptions are phenomenological ones at present. On the other hand, recently, Ma [34] has also proposed a model that can lead not only to the tribimaximal mixing (1), but also to the charged lepton mass relation (2) by assuming a symmetry $\Sigma(81)$. In Ma's $\Sigma(81)$ model, such an additional assumption except for the symmetry $\Sigma(81)$ has not been required. However, in his model, we need the somewhat unfamiliar and complicated symmetry $\Sigma(81)$. In contrast with the Ma model, in the present model we have adopted a familiar symmetry A_4 , and, instead, we have made some intuitive assumptions; (i) and (ii). Such an approach with phenomenological assumptions, at present, seems to be still useful for a future extension of the model rather than a rigid theoretical model.

In Sects. 5 and 6, a possible neutrino mass spectrum has been discussed by assuming $\langle \phi_1^u \rangle = \langle \phi_2^u \rangle = \langle \phi_3^u \rangle$, where ϕ_i^u belong to $\mathbf{3}$ of A_4 . By assuming the structure (42) of the right-handed neutrino Yukawa interaction, we can speculate the neutrino mass spectrum (46). The case $\alpha = \pi/6$ is interesting from the phenomenological point of view, because the case predicts the ratio $\Delta m_{\text{solar}}^2 / \Delta m_{\text{atm}}^2 = 0.0319$. However, the numerical predictions in Sect. 6 are not conclusive because the numerical prediction has needed some speculative assumptions.

In Sect. 7, a SUSY version for the Higgs potential of ϕ^d has been proposed. The essential idea is indebted to the Ma model based on the $\Sigma(81)$ symmetry [34].

The present model will give a suggestive hint on for seeking a more plausible model that leads to the tribimaximal mixing (1) and the charged lepton mass relation (2).

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